ARE PARTICULAR ABILITIES NECESSARY FOR PUPILS TO GAIN AN UNDERSTANDING OF THE ELEMENTARY AND SECONDARY MATHEMATICS AS USUALLY GIVEN AT THE PRESENT TIME.

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The student of mathematics in the elementary school and the high school is called upon to deal with three quite different and distinctly marked kinds of subject matter.

The first deals principally with the technical language of mathematics. This technical language is made up largely of pure conventions. There is no reasoning required, either inductive or deductive. All that is required is a knowledge of simply facts; for example, a mathematician uses a vertical cross to indicate multiplication. There is nothing in this sign itself that makes it indicate multiplication any more than division. It indicates multiplication simply because all those who use it have agreed that this shall be its meaning. Again, the expression $5a^3$ is written as it is written simply because people have agreed to write it so. If it had been agreed to write the third power of a taken five times as $3a^5$, we would have to learn it that way. The use and the meaning of the ordinary signs and symbols of mathematics come in the first instance under the head of things to be simply learned. Time was when the normal child could learn such things and the results reached in some schools today indicate that in some places it is being done today. It is, however, a lamentable fact that a large proportion of failures in mathematical study results from half learned mathematical speech. No particular intellectual faculty is required for the successful pursuit of this division of the work. There is nothing in it that requires any particular bulging of any particular part of the cranium. In fact, a healthy student might almost leave his cerebrum at home if his cerebellum were in good working order and still if he would get right down to hard work he might thoroughly master this phase of the subject of elementary and secondary mathematics. To the same

kind of subject matter belongs facility in performing ordinary operations in arithmetic and algebra and an adequate knowledge of a considerable class of algebraic expressions that are continually coming under observation. Just as we would expect the child to know the factors of 12 or 27 or 45 without thinking, without hesitation, so we would expect him to know the factors of $a^2 - b^2$, and of a large number of other type forms, and these are all of that class of matter that primarily require no reasoning. The child must know at sight the necessary fundamental things and must be able to do automatically the fundamental elementary cperations. There is no reason why every child of ordinary ability can not reach a high degree of perfection in this particular field.

The second class of questions has to do with things that may be apprehended by the senses or may be represented to the imagination. Most of the subject matter in the so-called practical arithmetic and in the geometry, plane and solid, is made up of this sort of matter. A student may see a bushel of potatoes, a cord of wood, a gallon of oil, a pound of sugar, and having once seen, he may be called on to think of larger or smaller quantities of these substances and may be expected to build up a mental picture which he can hold up before himself while he is determining relations or comparing proportions. Having once seen and examined critically a board and a post, it should be possible to construct in his mind a post and board fence conforming to any reasonable conditions. To do things of this sort requires only that the pupil shall have senses capable of perceiving, a memory capable of holding, and a power of visualization capable of bringing back a picture when required and of changing it at will. A serious defect in any one of these qualities of mind will make satisfactory results in elementary arithmetic and geometry impossible, but the normal child has all of these faculties, and, moreover, has infinite capacity of development. If a teacher is sufficiently wise to inquire just wherein the pupil is weak and shape instruction and training toward strengthening the weak spot, I can see no good reason why the normal child should not succeed fairly well in this division of the work.

The third kind of subject matter is less tangible, less concrete, includes the abstractions of mathematics and requires a develop-

ment of the reasoning faculties. Howiston in the introduction of one of the chapters of his analytical geometry makes essentially this statement: " A general locus is something which exists to abstract thought, but it can be neither drawn nor imagined." This third class of questions deals with things that can not be drawn or imagined, but which are still very real. The advanced arithmetic and the advanced algebra includes about all there is of this kind of material in the high school course. To successfully grapple with questions relating to this matter, the student must be thoroughly grounded in the knowledge and the art mentioned in the first class of questions, and, in addition, have a trained memory that will enable him to hold in mind the premises of his problem and the results of successive steps from the beginning to the end of the course of reasoning, and he must have a power of concentrating his attention that will enable him to carry on his course through several successive steps. Students will differ widely in their ability to successfully handle matters of this sort, just as they will differ widely in their ability to lift weights, to run a race, or to do any other thing, physical or intellectual, but I can see no reason why the normal child should not be able to make some progress in this direction.

So far I have given opinions only. I wish now to state some facts to sustain these opinions. As you all know, we have in this state a system of state-conducted examinations covering the high school field. You may or you may not be believers in examinations, but I think you will agree that if an examination is a fair test anywhere, it is in mathematics. I give in tabular form the results in all the high school mathematics in eight different schools in this state, covering a period of five years, and including a total of 19,249 papers written in ten different examinations. I have designated the schools by the letters, *A, B, C, D,* etc.:

	Examined.	Accepted.	Per Cent.
A	5,590	5,484	about 98
B	1,081	811	about 75
C	2,407	1,813	about 80
D	924	729	about 79
E	2,063	1,643	about 80
F	2,749	2,075	about 76
G	1,448	1,390	about 95
H	2,984	2,544	about 85

It will be noted that in school "*A*" there were 5,590 papers written; 5,484 were accepted. In school "*G*" 1,448 papers were written and 1,390 accepted. Now it must be admitted that if for a period of five years, a record of this sort can be maintained, no exceptional ability on the part of the pupil is required for success, unless we are to conclude that for some unquestioned and unquestionable reason, all the high school pupils in these two communities have for a period of five years possessed "these particular abilities." In all of the eight schools whose results are here tabulated, a fairly large proportion of all the students have given evidence of making a fairly successful record in mathematics. The result becomes still more conclusive when we remember that algebra and plane geometry are subjects required of all the students in the schools mentioned.

I am giving next a table showing the results in eight other schools, from which a different conclusion might have been reached if we had not the record given in the first table.

	Examined.	Accepted.	Per Cent.
I	3,819	2,015	about 53
J	618	409	about 65
K	795	493	about 62
L	917	551	about 60
M	631	349	about 55
N	1,797	781	about 43
O	318	130	about 41
P	920	614	about 67

It is very evident that the marked differences in the results in the two groups of schools that I have mentioned are not due to differences in the mental capacity of the children attending these schools. The difference may be due to any one of many causes or to a combination of many causes. It may be due to defective school organization, to the lack of a proper educational spirit in the school and in the community, to a lack of knowledge on the part of both teacher and student as to what constitutes a proper standard of excellence, and hence satisfaction with inferior results, all of which are only different forms of the results of poor teaching, for good teaching should lead to sound educational ideals not only among the pupils, but in the community at large, and should thereby establish reasonable educational standards.

Every teacher of wide experience has known pupils who thoroughly believed that it was impossible for them to make satisfactory progress in certain subjects. One student is very sure that he can not learn Latin. He has no aptitude for the study of foreign languages. Another is equally certain that while he makes good progress in Latin, he is utterly unable to deal satisfactorily with mathematics. A very large proportion of all the students are very certain that they can not do much with music and drawing and English composition. In my opinion, students who hold these ideas are in general mistaken, and the mistake has been due to poor teaching at some stage of their progress. I have known many pupils who were such failures in mathematics that they had become thoroughly convinced that it was impossible for them to make any progress in mathematical study, who, on a change of teachers, have been transformed into excellent students in mathematics. The same holds true of every other department of knowledge. Again, and from the same cause, a whole school may become obsessed by the notion that certain departments in the school curriculum are extremely difficult and other departments are so easy as to be classified under the well-known term "snap courses,"—courses in which credits may be easily obtained. The interesting fact about this is that while in school "*A*" mathematics may be considered as very difficult and Latin as very easy, in school "*B*" everybody passes in geometry, but only the most brilliant succeed in getting through the Latin. I have in mind a school where there was a wide range of election, in which every student desired to take geometry because every student in that school always passed geometry, while in the majority of schools of the same character, geometry was dreaded because of the difficulty in getting a passing mark. Suggestion plays a large part in the mental attitude of a pupil in school toward the subject matter in his course of study. Tradition also is a potent influence. Let it once become generally felt throughout a school that geometry is a very difficult subject to master and geometry becomes a thing to be dreaded, to be approached with fear and trembling, and very likely to be repeated before a passing mark is reached. In many colleges such a relief is felt when a class has once finished with trigonometry, analytical geometry or calculus, that the event is celebrated and

the old textbooks are burned in the celebration, but in one college it will be trigonometry, in another it will be analytical geometry, in another it will be calculus that has been handed down by tradition as the bugbear.

It is, of course, true that there are occasional individual examples of students whose mental development is abnormal to such an extent that the best teaching and the best environment fail to produce results in some subjects, but I believe there is not sufficient evidence to lead us to the conclusion that such cases are more common than cases of physical deformity.

It will not do for us teachers of mathematics to try to account for the inferior results of mathematical study in so many schools by laying it to the inferior mental capacity of the children. "What man has done, man can do." If in one group of schools, the students are uniformly successful in mathematical study and if in another group of schools under similar environment, the results of such instruction are unsatisfactory, there can be only one conclusion; that is, that in one case the teaching is better than in the other case. What children have done and are continually doing in one school, similar children can do in another school similarly situated, if they have equally good instruction. We will strengthen the weak places only when we are willing to recognize that "it is in ourselves that we are underlings."

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